



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

When  $V$  is a minimum,  $dV/d\theta=0$ , which condition is satisfied when  $\sin\theta=\sqrt{\frac{1}{3}}$ , i. e.,  $V=\frac{1}{2}(\pi r^3\sqrt{3})$ .

Also solved by R. A. Wells, J. E. Sanders, G. W. Greenwood, W. W. Landis, G. B. M. Zerr, E. L. Sherwood, J. Scheffer, and L. E. Newcomb.

## MECHANICS.

167. Proposed by EDWIN S. CRAWLEY, Ph.D., Professor of Mathematics in the University of Pennsylvania.

An anchor ring or torus is produced by the revolution of a circle of radius  $r$ , the center of the revolving circle describing a circle of radius  $R$ . A quadrant of the torus is cut by two planes through the center of the ring perpendicular to each other and perpendicular to the plane of revolution. Determine the limiting value of the ratio  $R:r$ , so that when the quadrant thus formed is placed with one of its ends in coincidence with a horizontal plane it will rest in that position.

I. Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

The center of gravity of the quadrant is at a distance  $d$  from the axis of revolution where  $d=(4R^2+r^2)/\pi R\sqrt{2}$  [Routh's *Statics* (second edition), Vol. I, ex. 4, p. 280].\* The limiting ratio of  $R$  to  $r$  is given by  $d/\sqrt{2}=R-r$ , whence  $4R^2+r^2=2\pi R(R-r)$ ; and  $R/r=2.9+$ .

II. Solution by B. F. FINKEL, Fellow in Mathematics, University of Pennsylvania, Philadelphia, Pa.

The equation of the surface of the ring is  $\rho=R\sin\phi\pm\sqrt{(r^2-R^2\cos^2\phi)}$ , when  $r$  is the radius of a cross-section of the ring and  $R$  the radius of the path of the center of gravity of the generating circle. Then

$$\bar{x}=2\frac{\int\sigma x dV}{\int dV},$$

where  $\sigma$  ( $=1$ , say) is the density  $dV$  and element of volume, and  $x$  the distance from the  $y$ -axis to this element of volume.

Then  $dV=\rho^2\sin\phi d\rho d\theta d\phi$ ,

$x=\rho\sin\phi\cos\theta$ , and

$\int dV=2\pi R\cdot\pi r/4=\frac{1}{2}\pi^2 r^2 R$ . Hence

$$\begin{aligned}\bar{x} &= \frac{4}{\pi^2 r^2 R} \int_{\cos^{-1}(r/R)}^{\frac{1}{2}\pi} \sin^2\phi d\phi \int_{R\sin\phi-\sqrt{(r^2-R^2\cos^2\phi)}}^{R\sin\phi+\sqrt{(r^2-R^2\cos^2\phi)}} \rho^3 d\rho \int_0^{\frac{1}{2}\pi} d\theta \\ &= \frac{8}{\pi^2 r^2} \int_{\cos^{-1}(r/R)}^{\frac{1}{2}\pi} [\sin^3\phi(2R^2\sin^2\phi+r^2-R^2)\sqrt{(r^2-R^2\cos^2\phi)}] d\phi.\end{aligned}$$

Let  $\cos\phi=(r/R)\cos\psi$ ; then the last equation reduces to

---

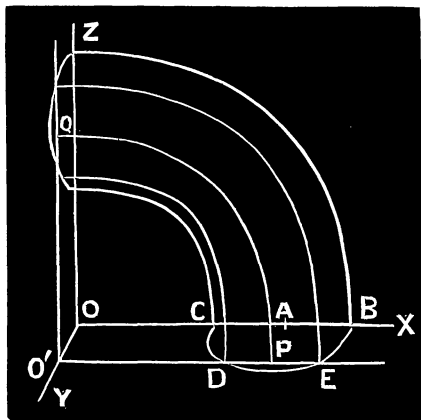
\*See also Bowser's *Analytical Mechanics*, 9th ed., p. 112.

$$\bar{x} = \frac{8}{\pi^2 R} \int_0^{\frac{1}{2}\pi} \left[ R^2 + r^2 - \frac{r^2}{R^2} (3R^2 + r^2) \cos^2 \psi + 2 \frac{r^2}{R^2} \cos^4 \psi \right] \sin^2 \psi d\psi = \frac{4R^2 + r^2}{2\pi R}$$

To insure stability, this must be greater than  $R - r$ . Hence, the limiting ratio is

$$\frac{R}{r^2} = \frac{\pi + \sqrt{(\pi^2 + 2\pi - 4)}}{2(\pi - 2)} = 2.9 + .$$

### III. Solution by the PROPOSER.



$OA = R$ ,  $AB = AE = r$ . Equation of section  $BEDC$  is

$$(x - R)^2 + y^2 = r^2.$$

$$\therefore O'E = R + \sqrt{(r^2 - y^2)},$$

$$O'D = R - \sqrt{(r^2 - y^2)}.$$

Take as element of volume the thread  $PQ$  whose length is  $\frac{1}{2}\pi x$  and cross-section  $dx dy$ . Then, if  $OY$  be taken as axis of moments the abscissa of the center of gravity of  $PQ$  is  $2x/\pi$ . Hence, the moment of the attraction of gravity upon the whole solid is

$$\begin{aligned} & 2 \int_0^r \int_{R-\sqrt{(r^2-y^2)}}^{R+\sqrt{(r^2-y^2)}} x^2 dx dy = 4R^2 \int_0^r \sqrt{(r^2-y^2)} dy + \frac{4}{3} \int_0^r (r^2-y^2)^{\frac{3}{2}} dy \\ & = 4r^2 R^2 \int_0^{\frac{1}{2}\pi} \cos^2 \theta d\theta + \frac{4}{3} r^4 \int_0^{\frac{1}{2}\pi} \cos^4 \theta d\theta = \pi r^2 R^2 + \frac{1}{4}\pi r^4 = \frac{\pi r^2}{4} (r^2 + 4R^2). \end{aligned}$$

The volume of the solid is  $\frac{1}{4}\pi r^2 \cdot 2\pi R = \frac{1}{2}\pi^2 r^2 R$ . Hence, the arm of the center of gravity is

$$\frac{\pi r^2}{4} (r^2 + 4R^2) \div \frac{1}{2}\pi^2 r^2 R = \frac{r^2 + 4R^2}{2\pi R}$$

If the solid is about to topple over the center of gravity is above  $O$ , and  $\frac{r^2 + 4R^2}{2\pi R} = R - r$ , from which

$$\frac{R}{r} = \frac{\pi \pm \sqrt{(\pi^2 + 2\pi - 4)}}{2(\pi - 2)}, \text{ or discarding the negative result,}$$

$$\frac{R}{r} = \frac{\pi + \sqrt{(\pi^2 + 2\pi - 4)}}{2(\pi - 2)} = 2.902, \text{ approximately.}$$

Also solved by S. A. Corey, Hiteman, Iowa.